

Neural Nonnegative Matrix Factorization for Hierarchical Multilayer Topic Modeling

Jamie Haddock

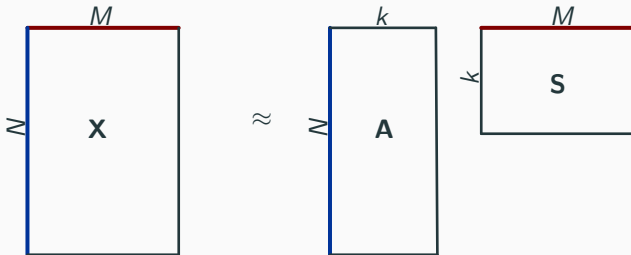
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Computational and Applied Mathematics
UCLA



joint with Mengdi Gao, Denali Molitor, Deanna Needell, Eli Sadovnik, Tyler Will, Runyu Zhang

Nonnegative Matrix Factorization (NMF)

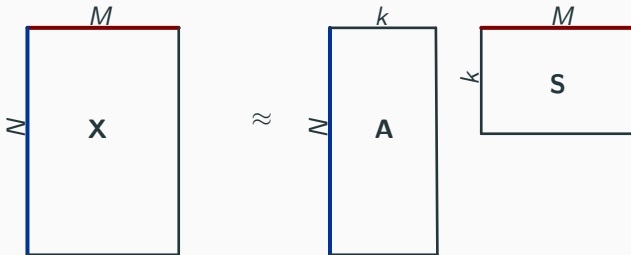


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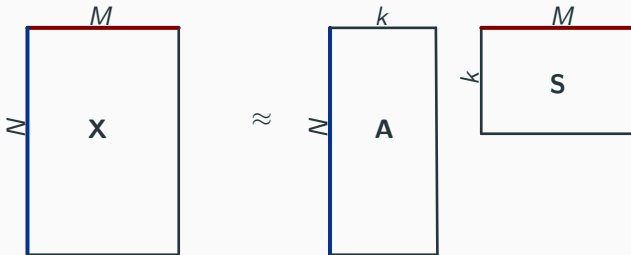
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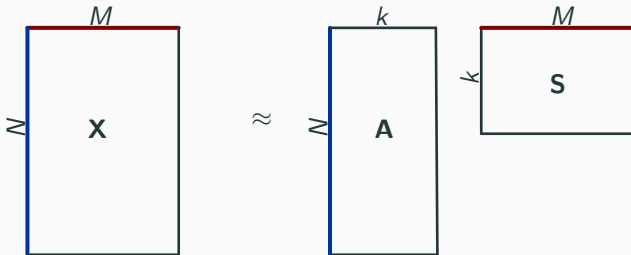
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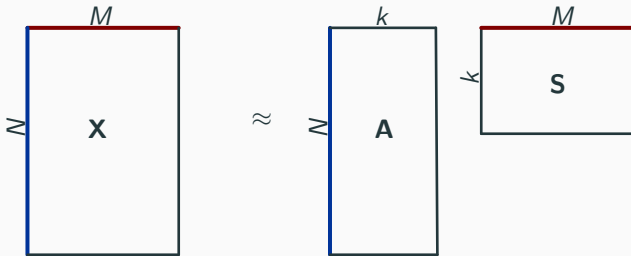
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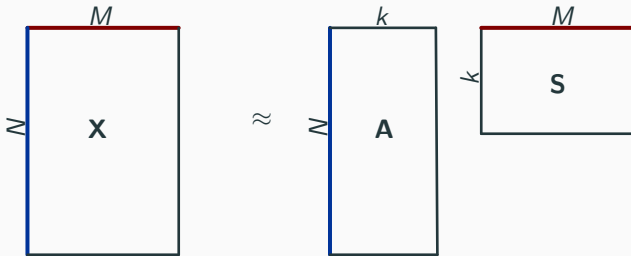
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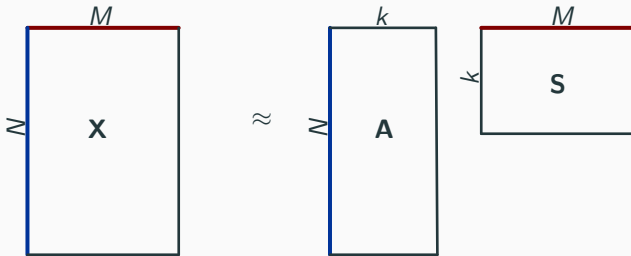
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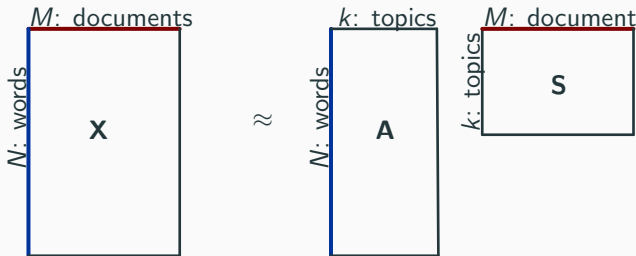
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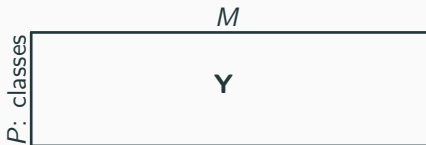
- ▷ low-rank approximation
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Methods:

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- ▷ many others

(Semi)supervised NMF

Goal: Incorporate known label information into problem.



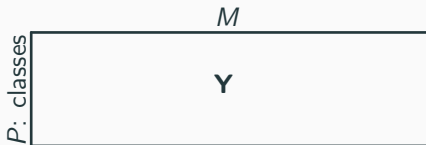
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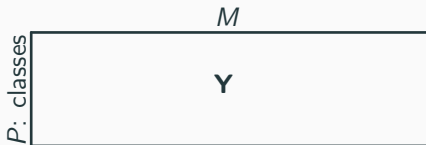
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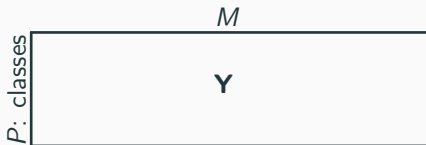
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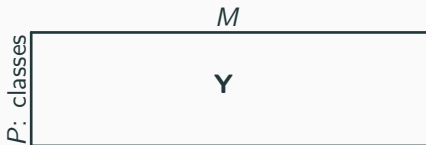
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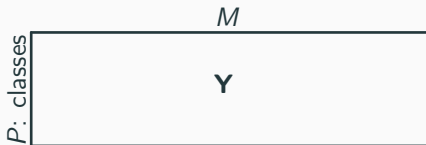
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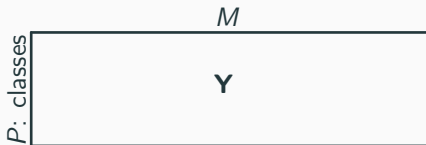
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- ▷ use of label information
- ▷ can extend multiplicative updates method to SSNMF

Hierarchical NMF

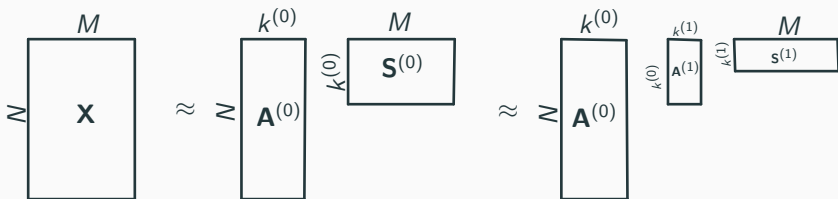
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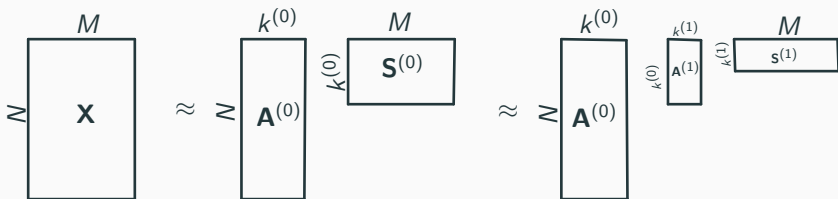
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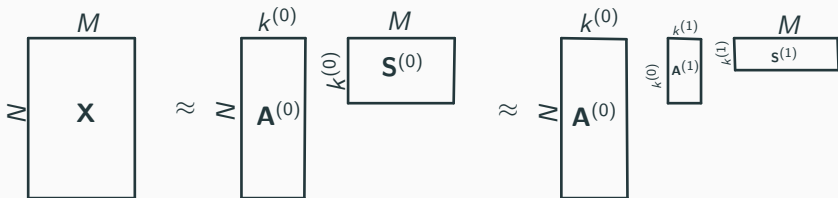
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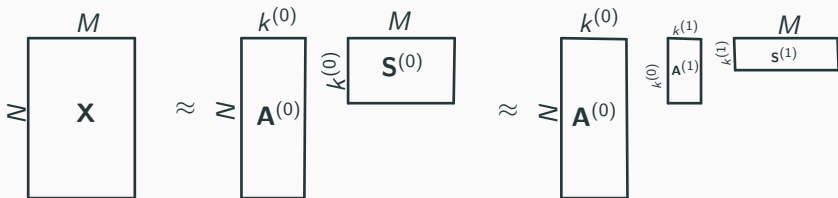
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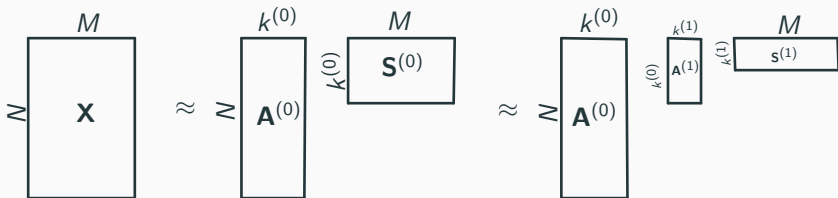
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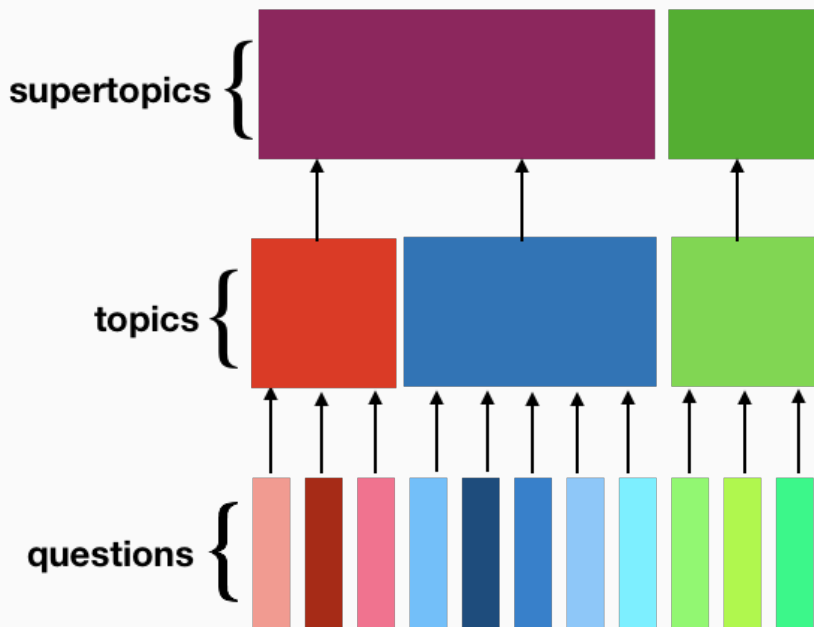
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Hierarchical NMF



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- ▷ [Sun, Nasrabadi, Tran '17]
 - similar method lacking nonnegativity constraints

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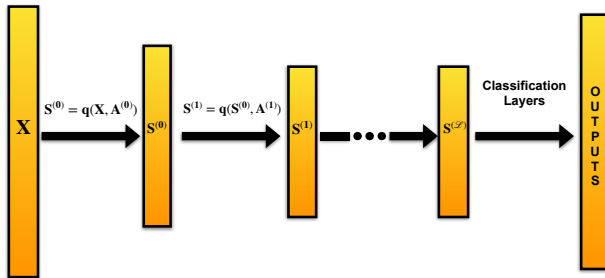
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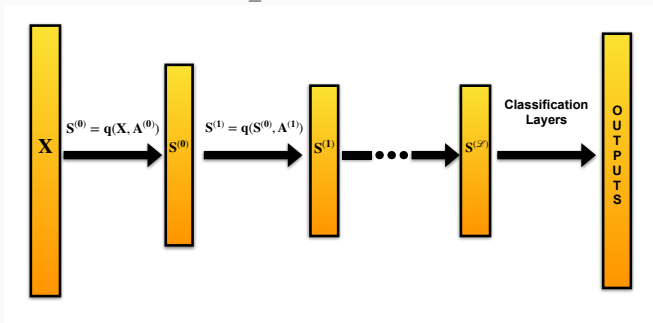
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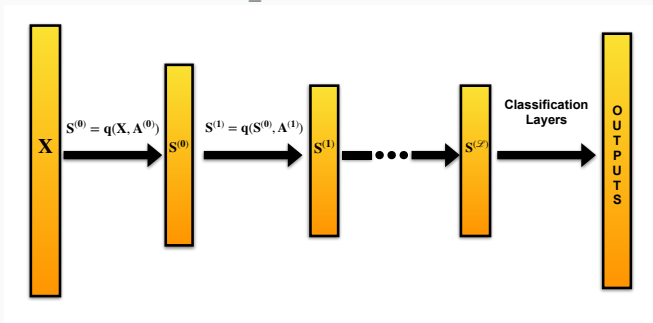


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- ▷ Can we compute derivatives and backpropagate?





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- ▷ Flexible to cost function (e.g., supervision).
- ▷ Backpropagate and update all **A** matrices simultaneously via GD or SGD.

Method 1 Neural NMF

Require: data matrix $\mathbf{X} \in \mathbb{R}^{N \times M}$, number of layers $\mathcal{L}$, step size γ , cost function C , initial matrices $\mathbf{A}^{(i)}$ for $i = 0, \dots, \mathcal{L}$

procedure FORWARDPROPAGATION($\mathbf{A}^{(0)}, \dots, \mathbf{A}^{(\mathcal{L})}$)

for $i := 0 \dots \mathcal{L}$ **do**

$\mathbf{S}^{(i)} \leftarrow q(\mathbf{A}^{(i)}, \mathbf{S}^{(i-1)})$

ForwardPropagation($\mathbf{A}^{(0)}, \dots, \mathbf{A}^{(\mathcal{L})}$)

while not converged **do**

for $i := 0 \dots \mathcal{L}$ **do**

$\mathbf{A}^{(i)} \leftarrow \mathbf{A}^{(i)} - \gamma * \frac{\partial C}{\partial \mathbf{A}^{(i)}}$

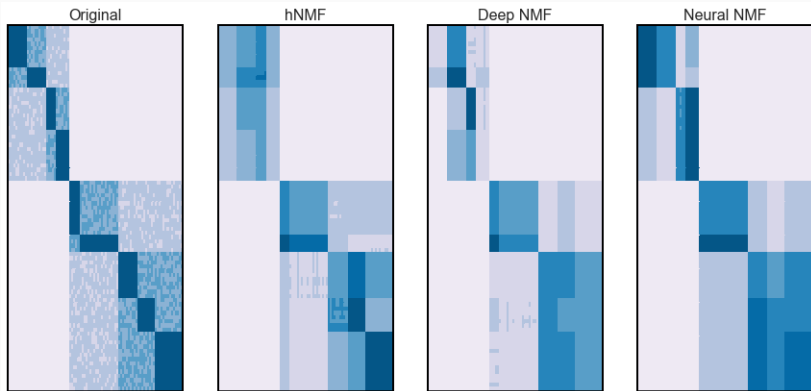
▷ Gradient descent

$\mathbf{A}^{(i)} \leftarrow \mathbf{A}_+^{(i)}$

▷ Project onto positive orthant

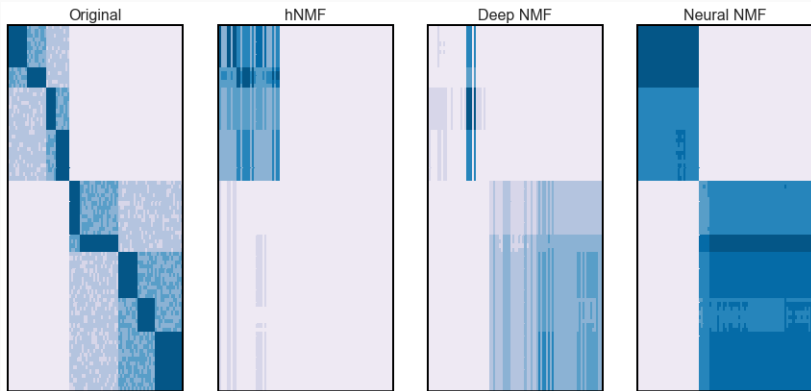
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Experimental Results



- ▷ unsupervised reconstruction with two-layer structure
($k^{(0)} = 9, k^{(1)} = 4$)

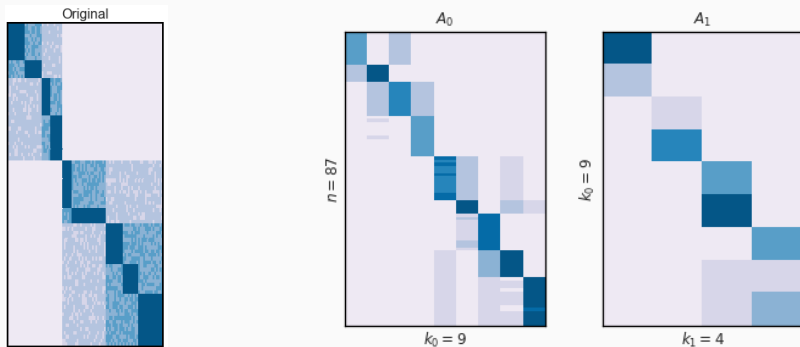
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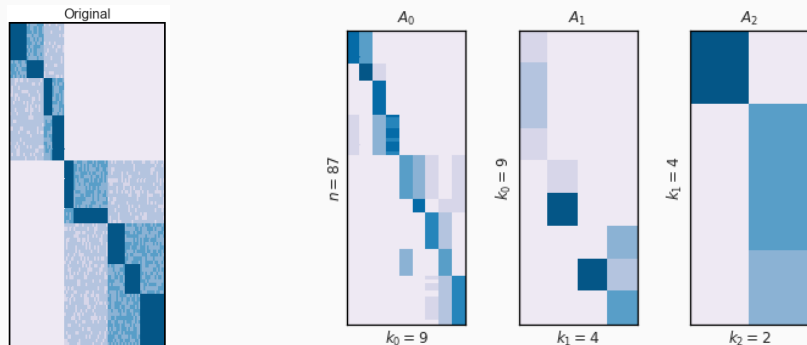
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Note that despite reconstruction error increasing as layers increase (since the final rank decreases), the topic structure can be resolved from the intermediate factorizations.



- ▷ semisupervised reconstruction (40% labels) with three-layer structure ($k^{(0)} = 9, k^{(1)} = 4, k^{(2)} = 2$)

Table 1: Reconstruction error / classification accuracy

	Layers	Hier. NMF	Deep NMF	Neural NMF
Unsuper.	1	0.053	0.031	0.029
	2	0.399	0.414	0.310
	3	0.860	0.838	0.492
Semisuper.	1	0.049 / 0.933	0.031 / 0.947	0.042 / 1
	2	0.374 / 0.926	0.394 / 0.911	0.305 / 1
	3	0.676 / 0.930	0.733 / 0.930	0.496 / 0.990
Supervised	1	0.052 / 0.960	0.042 / 0.962	0.042 / 1
	2	0.311 / 0.984	0.310 / 0.984	0.307 / 1
	3	0.495 / 1	0.494 / 1	0.498 / 1

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- ▷ extend to method for hierarchical nonnegative tensor factorization

Questions?

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